SPACE MECHANICS

Force is the action of one physical body on another, either through direct contact or through a distance. Gravity is an example of force acting through a distance, as are magnetism and the force between charged particles. The gravitational force between two masses m1 and m2 having a distance r between their centers is

$$\boldsymbol{F_g = G\frac{m_1 m_2}{r^2}}$$

This is **Newton's law of gravity**, in which G, the universal gravitational constant, has the value

$$G = 6.6742 \times 10^{-11} \ \frac{m^3}{Kg.s^2}$$

The force of a large mass (such as the earth) on a mass many orders of magnitude smaller (such as a person) is called weight, W. If the mass of the large object is M and that of the relatively tiny one is m, then the weight of the small body is

$$W = G\frac{Mm}{r^2}, \text{ But W = mg}$$

$$g = \frac{GM}{r^2}$$

Let g_0 represent the standard sea-level value of g, we get

$$\boldsymbol{g_0 = \frac{GM}{R_E^2}}$$

Where, $g_0 = 9.81\frac{m}{s^2}$

let z represent the distance above the earth's surface, so
that r = R_E+ z,

$$g = \frac{g_0}{\left(1 + \frac{z}{R_E}\right)^2}$$

GRAVITIONAL PARAMETER ($\boldsymbol{\mu}$)

$$\boldsymbol{\mu = G(m_1 + m_2)}$$

The units of $\boldsymbol{\mu}$ are cubic kilometres per square second.

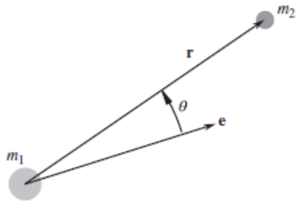
$$\mu = G(m_{Earth} + m_{Satelite})$$

$$m_{Earth} > m_{Satelite}$$

$$\mu_E = GM_{Earth} = g_0 R_E^2$$

$$\mu_E = 398600\ \frac{Km^3}{s^2}$$

ANGULAR MOMENTUM AND THE ORBIT FORMULAS



$$\boldsymbol{r = \frac{h^2}{\mu}\frac{1}{1 + eCos\theta}}$$

This is the orbit equation, and it defines the path of the body m2 around m1, relative to m1. Remember that m, h, and e are constants. Observe as well that there is no significance to negative values of eccentricity, that is, e≥ 0. Since the orbit equation describes conic sections, including ellipses, **it is a mathematical statement of Kepler's first law, namely, that the planets follow elliptical paths around the sun.** Two-body orbits are often referred to as Keplerian orbits.

Where,μ = Gravitional parameter
r = radius vector

θ = True anamoly (Angle between radius vector and Apse line)

e = eccentricity

h = the relative angular momentum of m_2 per unit mass, that is, the specific relative angular momentum. The units of h are square kilometres per second.

The angular momentum depends only on the **azimuthal (perpendicular or transverse) component of the relative velocity.**

$$\boldsymbol{h = rV_T}$$

KEPLER'S SECOND LAW

A line joining a planet and the sun sweeps out equal areas during equal intervals of time.

$$\frac{dA}{dt} = \frac{h}{2}$$

Where, $\frac{dA}{dt}$ is called the areal velocity

In other words,

$$h = rV_T \text{ = constant}$$

Note: The larger the value of r, i.e. farthest position of the mass, the velocity is minimum and the smaller the value of r, i.e. closest position of the mass, the velocity is maximum.

RADIAL AND AZIMUTHAL COMPONENTS OF VELOCITY

We know,

$$h = rV_T$$

therefore, $V_T = \frac{h}{r}$

Using Orbit equation in the above formula, we get,

$$V_T = \frac{\mu}{h}(1 + eCos\theta)$$

$$V_r = \frac{\mu}{h}eSin\theta$$

Where, $V = \sqrt{V_r^2 + V_T^2}$As $\theta = 0$, that m_2 comes closest to m_1 (r is the smallest) (unless e = 0, in which case the distance between m1 and m2 is constant). The point of closest approach lies on the apse line and is called Periapsis (Perigee). The distance r_P to Periapsis from the orbit equation is,

$$\boldsymbol{r_P = \frac{h^2}{\mu}\frac{1}{1+e}}$$

As $\theta = 180$, that m_2 comes farthest from m_1 (r is the largest) (unless e = 0, in which case the distance between m1 and m2 is constant). The point of farthest approach lies on the apse line and is called Apoapsis (Apogee). The distance r_a to apoapsis from the orbit equation is,

$$\boldsymbol{r_a = \frac{h^2}{\mu}\frac{1}{1-e}}$$

It is clear that the radial component of velocity ($V_r = 0$) is zero at Periapsis and Apoapsis.